Saturated Transfer Systems: Homework II

Mathcamp 2025

Problem 1 (recommended)**.** Describe $c(R)$ for all seven saturated transfer systems R on $\underline{2} \times \underline{2}$. Check that our computations of $|c^{-1}(A)|$ are correct in this case.

Problem 2 (recommended)**.** Use the recurrence to give explicit closed forms for $s(m, n)$ for $n = 1, 2, 3, 4$.

Problem 3 (optional)**.** Prove the following facts about the Stirling numbers of the second kind:

(a) $\left\{ {n \atop n} \right\} = 1$ for $n \geq 0$,

(b) $\left\{ {n \atop 0} \right\} = 1$ for $n \geq 1$,

(c) $\left\{ {n \atop k} \right\} = 0$ for $k > n$,

(d) $\left\{ {n \atop 1} \right\} = 1$ for $n \geq 1$,

(e) $\left\{ {n \atop 2} \right\} = 2^{n-1} - 1$ for $n \geq 2$,

(f) $\left\{ {n \atop n-1} \right\} = \binom{n}{2}$ for $n \geq 2$,

(g) $\left\{ {n+1 \atop k} \right\} = k\left\{ {n \atop k} \right\} + \left\{ {n \atop k-1} \right\}$ $0 < k < n$,

(h) $\left\{ {n \atop k} \right\} = \sum_{r_1 + \cdots + r_k = n-k} \prod_{i=1}^{k} i^{r_i}$, where the sum is over all nonnegative ordered partitions of $n - k$ (hint: what is i^{r_i} counting?), and

(i) $\left\{ {n \atop k} \right\} = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} j^n$ (hint: use inclusion-exclusion.)